A GRAPHICAL APPROACH TO PRECALCULUS WITH LIMITS

HORNSBY
LIAL
ROCKSWOLD

7TH EDITION

Pearson

Our Unifying Approach to Functions

Our approach to studying the functions of algebra allows students to make connections between graphs of functions, their associated equations and inequalities, and related applications. To demonstrate this four-part process using a quadratic function (Chapter 3), consider the following illustrations.

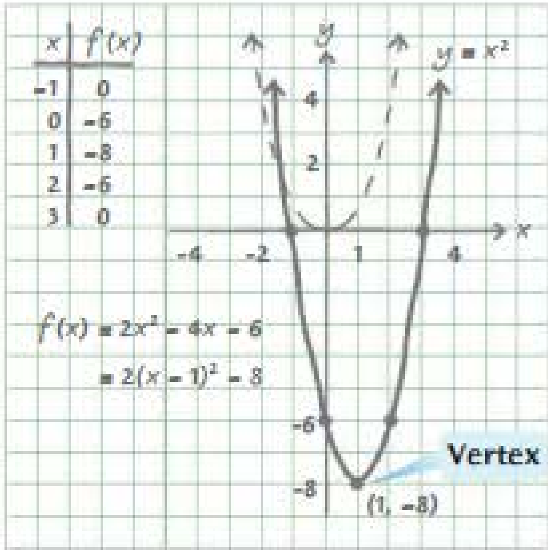
1 Examine the nature of the graph.

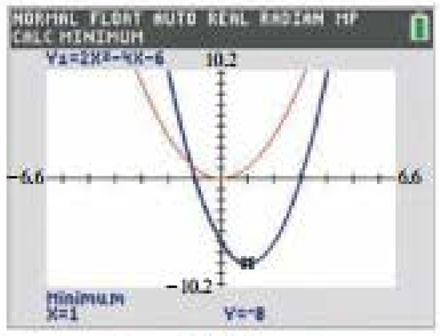
ILLUSTRATION: Graph $f(x) = 2x^2 - 4x - 6$.

Solution Because the function is quadratic, its graph is a parabola. By completing the square, it can be written in the form

$$f(x) = 2(x - 1)^2 - 8.$$

When $Y_1 = 2X^2 - 4X - 6$ is compared to $Y_2 = X^2$, its graph is shifted horizontally 1 unit to the right, stretched by a factor of 2, and shifted vertically 8 units down. Its vertex has coordinates $(1, -8)$, and the axis of symmetry has equation $x = 1$. The domain is $(-\infty, \infty)$, and the range is $[-8, \infty)$.





$Y_2 = X^2$ is in red.

2 Solve a typical equation analytically and graphically.

ILLUSTRATION: Solve the equation $2x^2 - 4x - 6 = 0$.

Analytic Solution

$$2x^2 - 4x - 6 = 0$$

$$x^2 - 2x - 3 = 0 \quad \text{Divide by 2.}$$

$$(x + 1)(x - 3) = 0 \quad \text{Factor.}$$

$$x + 1 = 0 \quad \text{or} \quad x - 3 = 0 \quad \text{Zero-product property}$$

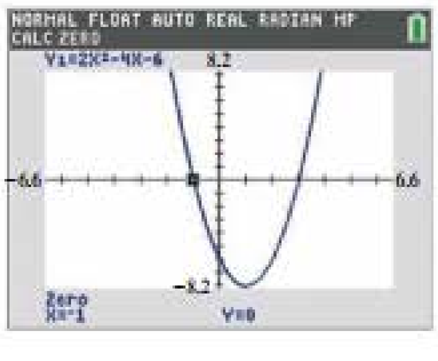
$$x = -1 \quad \text{or} \quad x = 3 \quad \text{Solve each equation.}$$

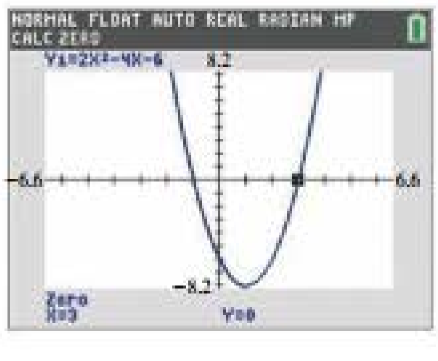
Check by substituting the values -1 and 3 for x in the original equation.

The solution set is $\{-1, 3\}$.

Graphing Calculator Solution

Using the x-intercept method, we find that the zeros of $Y_1 = 2X^2 - 4X - 6$ are the solutions of the equation.

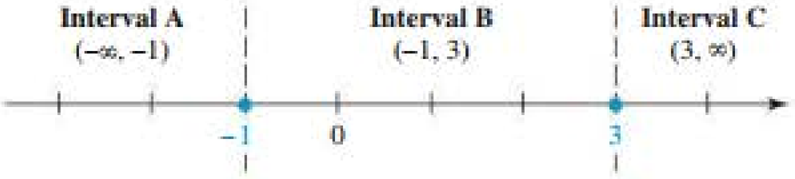




3 Solve the related inequality analytically and graphically.

ILLUSTRATION: Solve the inequality $2x^2 - 4x - 6 \le 0$.

Solution Divide a number line into intervals determined by the zeros of $f(x) = 2x^2 - 4x - 6$, (found in Illustration 2), which are -1 and 3. Choose a test value from each interval to identify values for which $f(x) \le 0$.



Interval	Test Value x	Is $f(x) = 2x^2 - 4x - 6 \le 0$ True or False?
A: $(-\infty, -1)$	-2	$f(-2) = 2(-2)^2 - 4(-2) - 6 \le 0$? $10 \le 0$ False
B: $(-1, 3)$	0	$f(0) = 2(0)^2 - 4(0) - 6 \le 0$? $-6 \le 0$ True
C: $(3, \infty)$	4	$f(4) = 2(4)^2 - 4(4) - 6 \le 0$? $10 \le 0$ False

From the table, the polynomial $2x^2 - 4x - 6$ is *negative* or *zero* on the interval $[-1, 3]$. The calculator graph in Illustration 2 supports this solution, since the graph lies *on* or *below* the x-axis on this interval.

4 Apply analytic and graphical methods to solve an application of that class of function.

ILLUSTRATION: If an object is projected directly upward from the ground with an initial velocity of 64 feet per second, then (neglecting air resistance) the height of the object x seconds after it is projected is modeled by

$$s(x) = -16x^2 + 64x,$$

where $s(x)$ is in feet. After how many seconds does it reach a height of 28 feet?

Analytic Solution

We solve the equation $s(x) = 28$.

$$s(x) = -16x^2 + 64x$$

$$28 = -16x^2 + 64x \quad \text{Let } s(x) = 28.$$

$$16x^2 - 64x + 28 = 0 \quad \text{Standard form}$$

$$4x^2 - 16x + 7 = 0 \quad \text{Divide by 4.}$$

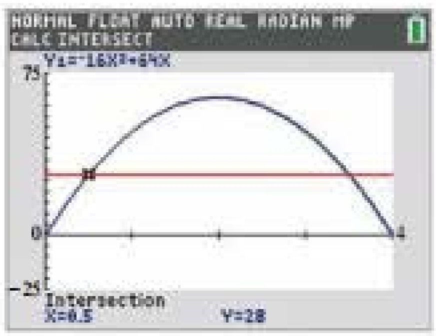
$$(2x - 1)(2x - 7) = 0 \quad \text{Factor.}$$

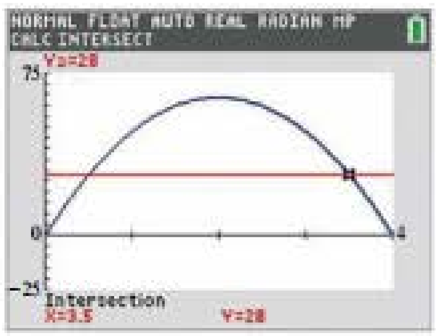
$$x = 0.5 \quad \text{or} \quad x = 3.5 \quad \text{Zero-product property}$$

The object reaches a height of 28 feet twice, at 0.5 second (on its way up) and at 3.5 seconds (on its way down).

Graphing Calculator Solution

Using the intersection-of-graphs method, we see that the graphs of $Y_1 = -16X^2 + 64X$ and $Y_2 = 28$ intersect at points whose coordinates are (0.5, 28) and (3.5, 28), confirming our analytic answer.





A GRAPHICAL APPROACH TO PRECALCULUS WITH LIMITS: A UNIT CIRCLE APPROACH

SEVENTH EDITION

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In memory of my mentor and friend,
Earl Swokowski,
who passed before his time
GR

Foreword

The first edition of *A Graphical Approach to Precalculus with Limits: A Unit Circle Approach* was published in 1996. Our experience was that the usual order in which the standard topics were covered did not foster students' understanding of the interrelationships among graphs, equations, and inequalities. The table of contents for typical texts did not allow for maximum effectiveness in implementing our philosophy because graphs were not covered early enough in the course. Thus, we reorganized the standard topics with early introduction to the graphs of functions, followed by solutions of equations, inequalities, and applications. Although the material is reorganized, ***we still cover all traditional topics and skills.*** The underlying theme was, and still is, to illustrate how the graph of a typical function can be used to support the solutions of equations and associated inequalities involving the function.

Using linear functions in Chapter 1 to introduce the approach that follows in later chapters, we apply a four-step process of analysis.

1. **We examine the nature of the graph of the function,** using both hand-drawn and calculator-generated versions. Domain and range are established, and any further characteristics are discussed.
2. **We solve equations analytically,** using the standard methods. Then we support our solutions graphically, using the *intersection-of-graphs method* and the *x-intercept method* (pages 54–55).
3. **We solve the associated inequalities analytically,** again using standard methodology, supporting their solutions graphically as well.
4. **We apply analytic and graphical methods to modeling and traditional applications** involving the class of function under consideration.

After this procedure has been initially established for linear functions, we apply it to absolute value, quadratic, higher-degree polynomial, rational, root, exponential, logarithmic, and trigonometric functions in later chapters. The chapter on systems of equations ties in the concept of solving systems with the aforementioned intersection-of-graphs method of solving equations.

This presentation provides a sound pedagogical basis. Because today's students rely on visual learning more than ever, the use of graphs promotes student understanding in a manner that might not occur if only analytic approaches were used. It allows the student the opportunity to see how the graph of a function is related to equations and inequalities involving that function. Students are presented with the same approach over and over, and they come to realize that the *type* of function f defined by $y = f(x)$ under consideration does not matter when providing graphical support. For example, using the x-intercept method, the student sees that x-values of x-intercepts of the graph of $y = f(x)$ correspond to real solutions of the equation $f(x) = 0$, x-values of points above the x-axis correspond to solutions of $f(x) > 0$, and x-values of points below the x-axis correspond to solutions of $f(x) < 0$.

The final result, in conjunction with the entire package of learning tools provided by Pearson, is a course that covers the standard topics of precalculus. It is developed in such a way that graphs are seen as pictures that can be used to interpret analytic results. We hope that you will enjoy teaching this course, and that your students will come away with an appreciation of the impact and importance of our approach in the study of precalculus.

John Hornsby
Gary Rockswold

Contents

Preface

Although *A Graphical Approach to Precalculus with Limits: A Unit Circle Approach* has evolved significantly from earlier editions, it retains the strengths of those editions and provides new and relevant opportunities for students and instructors alike. We realize that today's classroom experience is evolving and that technology-based teaching and learning aids have become essential to address the ever-changing needs of instructors and students. As a result, we have worked to provide support for all classroom types—traditional, hybrid, and online. In the seventh edition, text and online materials are more tightly integrated than ever before. This enhances flexibility and ease of use for instructors and increases success for students. See pages xviii–xx for descriptions of these materials.

This text incorporates an open design, helpful features, careful explanations of topics, and a comprehensive package of supplements and study aids. We continue to offer an *Annotated Instructor's Edition,* in which answers to both even- and odd-numbered exercises are provided either beside the exercises (if space permits) or in the back of the text for the instructor.

A Graphical Approach to Precalculus with Limits: A Unit Circle Approach was one of the first texts to reorganize the typical table of contents to maximize the use of graphs to support solutions of equations and inequalities. It maintains its unique table of contents and functions-based approach (as outlined in the Foreword and in front of the text) and includes additional components to build skills, address critical thinking, and give students a wealth of opportunities to solve applications and make use of technology to support traditional analytic solutions.

This text is part of a series that also includes the following titles:

- *A Graphical Approach to College Algebra, Seventh Edition,* by Hornsby, Lial, and Rockswold
- *A Graphical Approach to Algebra and Trigonometry, Seventh Edition,* by Hornsby, Lial, and Rockswold

The book is written to accommodate students who have access to graphing calculators. We have chosen to use screens from the TI-84 Plus C emulator. However, we do not include specific keystroke instructions because of the wide variety of models available. Students should refer to the guides provided with their calculators for specific information.

New to This Edition

There are many places in the text where we have refined individual presentations and added examples, exercises, and applications based on reviewer feedback. The changes you may notice include the following:

- A NEW recurring feature is titled **Unifying Functions.** Following discussion of each of the important functions (for example, Unifying Linear Functions on page 67), we present a concise summary that covers Analyzing the Graph, Solving an Equation, Solving an Inequality, and Solving an Application. This feature reinforces the general approach of the text. Accompanying **videos** are embedded in the eText and **assessment questions** are available in MyLab Math.

- **Applications** have been updated throughout the text in such areas as organic food sales, video-on-demand, active Twitter users, worldwide WhatsApp usage, U.S. Snapchat users, top social networks, wearable technology, fast-food restaurant and advertising revenue, world records in track, college enrollment, poverty-level income cutoffs, health care expenditures, online sales, airport runway designations, online gaming revenue, population, vehicle sales, and pollutant emissions.
- Graphing calculator screens have been updated using the **TI-84 Plus C** emulator, often employing pedagogical color.
- **Chapter 1** New *Technology Note* explaining the equivalence of different function notation styles; updated examples throughout.
- **Chapter 2** More discussion about the constant function; more exercises that determine whether a function is odd or even; additional discussion, examples, and exercises about the order in which to apply combinations of transformations; the difference quotient and average rate of change; composite functions and their domains; additional examples of graphical solutions to equations and inequalities; a new subsection on error tolerances with examples and exercises; more graphing of absolute value functions by hand; a new example and exercises related to piecewise-defined functions.

  (Note: Chapter 3 from the previous edition has been divided into two chapters at the suggestion of reviewers. In the seventh edition, Chapter 3 consists of former Sections 3.1–3.4, and Chapter 4 consists of former Sections 3.5–3.8.)
- **Chapter 3** Additional exercises on quotients of complex numbers; a new subsection on "A Quadratic Relation: The Circle" (this gives the instructor the option to cover circles and completing the square to find the center and radius earlier than in previous editions); new examples and exercises have been added throughout; exercises on complex numbers and exercises on circles have been added to the end-of-chapter Summary and Test.
- **Chapter 4** Introduces the terms *upper bound* and *lower bound;* updated examples and exercises appear throughout; additional exercises on polynomial function behavior.
- **Chapter 5** A new example about analyzing graphs of rational functions; new exercises where asymptotes are described using limit notation; new examples and exercises where rational functions are graphed by hand; new examples in which rational inequalities are solved; additional discussion about graphing circles with a calculator; new exercises that involve solving radical inequalities.
- **Chapter 6** Applications of logarithms with bases other than e and 10 have been supplemented with discussion of modern calculator capabilities of computing them directly (the change-of-base rule is still covered); a new example on modeling the number of monthly active Twitter accounts; new discussion, example, and exercises on modeling with logistic functions.
- **Chapter 7** Additional exercises that provide practice in solving systems of equations; more investment examples and applications; new coverage of systems that have infinitely many solutions; many new examples and exercises in which systems are solved by hand using row transformations; more discussion and exercises that involve solving rational inequalities; a new example and exercises about partial fraction decomposition.
- **Chapter 8** An example using parametric equations for an object in motion has been expanded; new exercises for parametric graphs have been included.

- **Chapter 9** A new subsection that discusses the real life application of airport runways, including an example and exercises; new examples of finding trigonometric function values using reference angles, finding angle measures by hand, evaluating trigonometric functions involving triangles, evaluating circular functions, and analyzing damped harmonic motion; new and additional exercises related to clock hands and angular velocity, rationalizing the denominator, reference angles, evaluating inverse trigonometric functions, solving triangles, finding exact values of trigonometric functions, writing equations of given graphs, modeling real data, damped harmonic motion, music, and function values as lengths of line segments.
- **Chapter 10** A complete revision of the material covering the inverse cotangent, inverse secant, and inverse cosecant functions, including new Function Capsules; new Concept Check exercises, including matching exercises; additional exercises related to solving trigonometric equations.
- **Chapter 11** New Concept Check exercises and a new Discussing Concept feature; rewording to make identities and trigonometry, in general, more accessible; additional graphing calculator explanations and solutions involving polar coordinates; new exercises involving whether a triangle with the given conditions exists; additional exercises involving plotting complex numbers in the complex plane.
- **Chapter 12** New exercises in solving inequalities that involve both sequences and series; new examples and exercises about mathematical induction; more discussion and exercises about odds in gambling.
- **Chapter 13** New examples covering limits at points of discontinuity, limits of square root functions, finding the equation of a tangent line and graphing it, interpreting the derivative in an application; new exercises involving limits at points of discontinuity, rational and trigonometric functions, both full and one-sided limits of square root, logarithmic, absolute value, and exponential functions; new exercises requiring equations of tangent lines, interpretation of the derivative, and evaluating a definite integral geometrically; two new Discussing Concepts.
- **Chapter R (formerly called "Reference," now called "Review")** A section on Review of Sets has been added.

Features

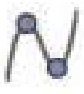
We are pleased to offer the following enhanced features:

Chapter Openers Chapter openers provide a chapter outline and a brief discussion related to the chapter content.

Enhanced Examples We have replaced some examples and have included many new examples in this edition. We have also polished solutions and incorporated more explanatory comments and pointers.

Hand-Drawn Graphs We have incorporated many graphs featuring a "hand-drawn" style that simulates how a student might actually sketch a graph on grid paper. Accompanying videos are available in the MyLab Math multimedia library.

Dual-Solution Format Selected examples continue to provide side-by-side analytic and graphing calculator solutions, to connect traditional analytic methods for solving problems with graphical methods of solution or support. NEW! Embedded links in the eText enable students to launch a pop-up GeoGebra graphing calculator for these examples (see icon to left).



Pointers Comments with pointers (bubbles) provide students with on-the-spot explanations, reminders, and warnings about common pitfalls.

Highlighted Section and Figure References Within the text we use boldface type when referring to numbered sections and exercises (e.g., **Section 2.1, Exercises 15–20**). We also use a corresponding font when referring to numbered figures (e.g., **FIGURE 1**). We thank Gerald M. Kiser of Woodbury (New Jersey) High School for this latter suggestion.

Figures and Photos Today's students are more visually oriented than ever. As a result, we have made a concerted effort to provide more figures, diagrams, tables, and graphs, including the "hand-drawn" style of graphs, whenever possible. And we often provide photos to accompany applications in examples and exercises.

Function Capsules These special boxes offer a comprehensive, visual introduction to each class of function and serve as an excellent resource for reference and review. Each capsule includes traditional and calculator graphs and a calculator table of values, as well as the domain, range, and other specific information about the function. Abbreviated versions of function capsules are provided on the inside back cover of the text.

What Went Wrong? This popular feature explores errors that students often make when using graphing technology and provides an avenue for instructors to highlight and discuss such errors. Answers are included on the same page as the "What Went Wrong?" boxes. Accompanying **videos** are available in the MyLab Math multimedia library.

Cautions and Notes These features warn students of common errors and emphasize important ideas throughout the exposition.

Looking Ahead to Calculus These margin notes provide glimpses into how the algebraic topics the students are currently studying are used in calculus.

Algebra Reviews This feature, which appears in the margin of the text, provides "just in time" review by indicating where students can find additional help with important topics from algebra.

Technology Notes Also appearing in the margin, these notes provide tips on how to use graphing calculators more effectively.

Discussing Concepts These activities appear within the exposition or in the margins and offer material on important concepts for instructors and students to investigate or discuss in class.

Exercise Sets We have taken special care to respond to the suggestions of users and reviewers and have added hundreds of new exercises to this edition on the basis of their feedback. The text continues to provide students with ample opportunities to practice, apply, connect, and extend concepts and skills. We have included writing exercises as well as multiple-choice, matching, true/false, and completion problems. Exercises marked CONCEPT CHECK focus on mathematical thinking and conceptual understanding, while those marked CHECKING ANALYTIC SKILLS are intended to be solved *without the use of a calculator*.

Relating Concepts These groups of exercises appear in selected exercise sets. They link topics together and highlight relationships among various concepts and skills. All answers to these problems appear in the answer section at the back of the student text.

Reviewing Basic Concepts These sets of exercises appear every two or three sections and give students an opportunity to review and check their understanding of the material in preceding sections. All answers to these problems are included in the answer section.

Chapter Review Material One of the most popular features of the text, each end-of-chapter Summary features a section-by-section list of Key Terms and Symbols, in addition to Key Concepts. A comprehensive set of Chapter Review Exercises and a Chapter Test are also included.

Acknowledgments

Previous editions of this text were published after thousands of hours of work, not only by the authors, but also by reviewers, instructors, students, answer checkers, and editors. To these individuals and to all those who have worked in some way on this text over the years, we are most grateful for your contributions. We could not have done it without you.

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John Hornsby
Gary Rockswold

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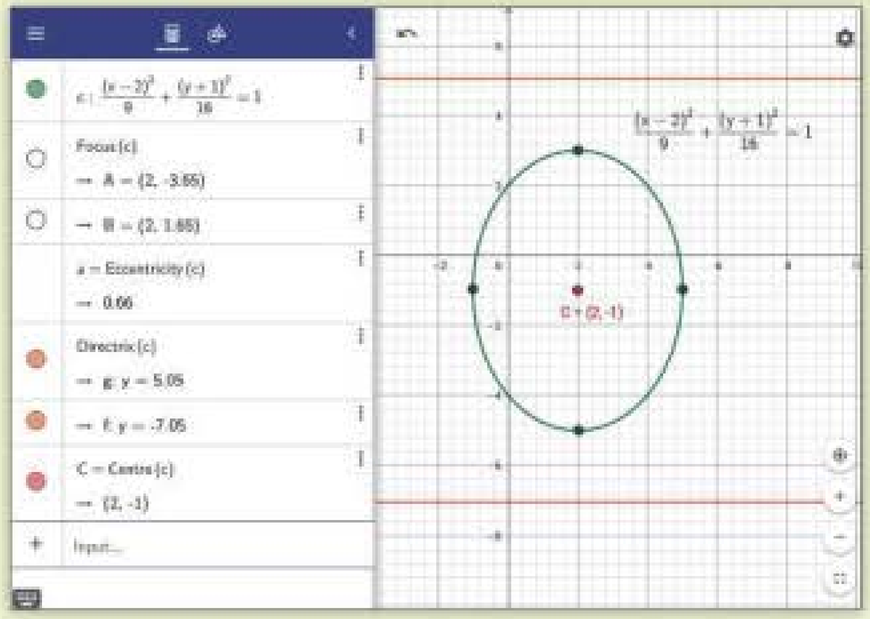
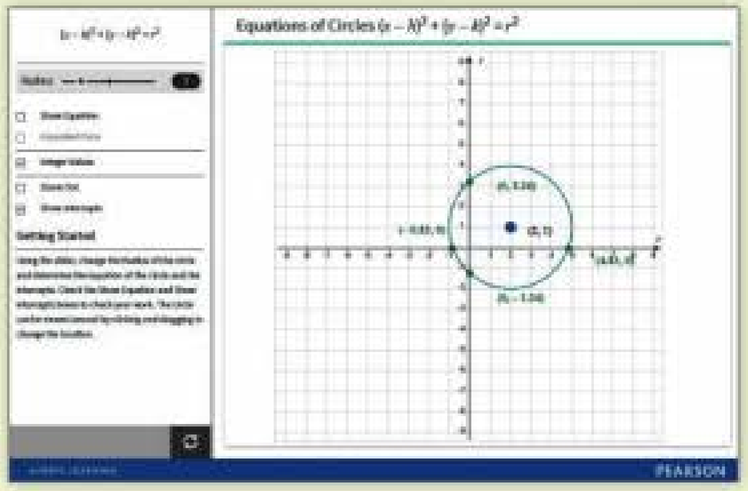
New! GeoGebra® Graphing Calculator and Tutorials

support Hornsby's graphical approach. GeoGebra, an online graphing utility, is integrated into the MyLab Math course. Look for the GeoGebra icon within the eText to open a pop-up version, deploy in a new browser tab from Graphing Resources, or download the free app to use while doing homework. Interactive, self-checking tutorials make it easy to get started with this dynamic tool.





Guided Visualizations

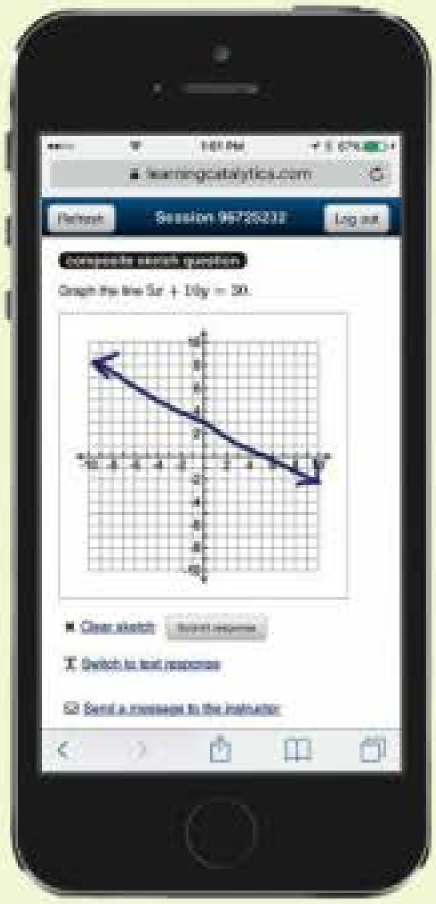
bring mathematical concepts to life, helping students visualize the concepts through directed explorations and purposeful manipulation. Guided Visualizations can be assigned in MyLab Math to encourage active learning, critical thinking, and conceptual understanding.

Student Motivation

Learning Catalytics

generate class discussion, guide your lecture, and promote peer-to-peer learning with real-time analytics. MyLab Math now provides Learning Catalytics—an interactive student response tool that uses students' smartphones, tablets, or laptops to engage them in more sophisticated tasks and thinking. Instructors, you can:

- Pose a variety of open-ended questions that help your students develop critical thinking skills.
- Monitor responses to find out where students are struggling.
- Use real-time data to adjust your instructional strategy and try other ways of engaging your students during class.
- Manage student interactions by automatically grouping students for discussion, teamwork, and peer-to-peer learning.



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Resources for Success

Instructor Resources

These additional resources can be downloaded from www.pearson.com or from within your MyLab Math course.

Annotated Instructor's Edition

This edition provides answers beside the text where possible for quick reference and in an answer section at the back of the book for all others.

Instructor's Solutions Manual

By David Atwood, *Rochester Community and Technical College*

This manual provides complete solutions to all text exercises.

TestGen®

TestGen (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

PowerPoint® Lecture Slides

The PowerPoint lecture slides feature presentations written and designed specifically for this text, including figures and examples from the text.

Instructor's Testing Manual

By David Atwood, *Rochester Community and Technical College*

This testing manual contains four alternative tests for each chapter and corresponding answer keys.

Student Activities Guides

By Susan Fife

This guide contains answers to *Precalculus Activities using WolframAlpha™* and *Engaging Algebra: Activities that Create Connections*.

Sample Assignments in MyLab Math

Enhanced Sample Assignments, created by the authors, make course set-up easier by giving instructors a starting point for each chapter. Each assignment, handpicked by the author team to align with this text, includes a thoughtful mix of question types (e.g., conceptual, skills, etc.) specific to that topic. Each assignment includes the newest MyLab Math question types including video assessments and guided visualizations.

Learning Catalytics

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Student's Solutions Manual

By David Atwood, *Rochester Community and Technical College*

This manual provides detailed solutions to odd-numbered Section and Chapter Review Exercises, as well as to all Relating Concepts, Reviewing Basic Concepts, and Chapter Test problems.

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Example Solution videos provide comprehensive coverage of key topics in the text in an engaging format. **Unifying Functions** videos summarize the 4-step solving process featured throughout the textbook for each of the major classes of functions. All videos are assignable in MyLab Math and available in the Multimedia Library.

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pearson.com/mylab/math

1

Linear Functions, Equations, and Inequalities

A map is an example of a plane in which points can be located with rectangular coordinates, such as those provided by the Global Positioning System (GPS). Two cities can be represented by points on the map, and the shortest distance between them is the measure of the line segment joining them. (This is the source of the saying "as the crow flies.") The segment is a portion of the unique straight line on which the points lie. These and other concepts associated with lines are fundamental to the study of linear functions, the subject of this chapter.

CHAPTER OUTLINE

1.1 Real Numbers and the Rectangular Coordinate System

Sets of Real Numbers • The Rectangular Coordinate System • Viewing Windows • Approximations of Real Numbers • Distance and Midpoint Formulas

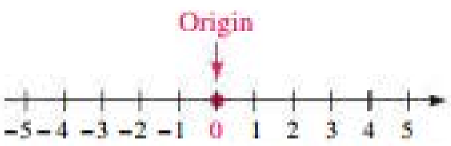
Sets of Real Numbers

In the first two chapters of this text, we study *real numbers.* **Real numbers** are those that can be represented by points on a *number line* and can be expressed as decimal numerals. There are several important sets of numbers included within the real number system. We use set notation to describe them. The *elements* of the set are either listed or described using *set-builder* notation.

NOTE Set-builder notation is used in the following table in the final three sets described. We read the description for rational numbers as "the set of all p divided by q such that p and q are integers and q is not equal to 0."

Sets of Numbers

Set	Description	Examples
Natural Numbers	$\{1, 2, 3, 4, \ldots\}$	$1, 45, 127, 10^3$
Whole Numbers	$\{0, 1, 2, 3, 4, \ldots\}$	$0, 86, 345, 2^3$
Integers	$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$	$0, -5, -10^2, 99$
Rational Numbers	$\left\{\frac{p}{q} \middle\vert p \text{ and } q \text{ are integers}, q \neq 0\right\}$	$0, -\frac{5}{6}, -2, \frac{22}{7}, 0.5$
Irrational Numbers	$\{x \vert x \text{ is not rational}\}$	$\sqrt{2}, \pi, -\sqrt[3]{7}$
Real Numbers	$\{x \vert x \text{ is a decimal number}\}$	$-\sqrt{6}, \pi, \frac{2}{3}, \sqrt{45}, 0.41$



Real number line with the origin plotted
FIGURE 1

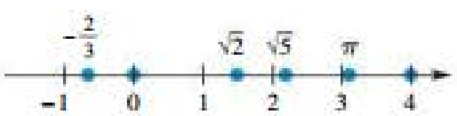
Whole numbers consist of the **natural numbers** and **0. Integers** consist of the whole numbers and the negatives of the natural numbers. The result of dividing two integers (with a nonzero divisor) is a **rational number,** or *fraction*. Rational numbers include the integers. For example, the integer -3 is a rational number because it can be written as $\frac{-3}{1}$. Every rational number can be written as a repeating or terminating decimal. For example, $0.\overline{6} = 0.66666\ldots$ represents the rational number $\frac{2}{3}$.

Real numbers consist of both rational and irrational numbers and can be shown pictorially—that is, *graphed*—on a **number line.** The point on a number line corresponding to 0 is called the **origin.** See FIGURE 1. Numbers that lie to the right of 0 are *positive* numbers, and those that lie to the left of 0 are *negative* numbers. The number 0 is neither positive nor negative.

Graph of the set of real numbers
FIGURE 2

Every real number corresponds to one and only one point on the number line, and each point corresponds to one and only one real number. This correspondence is called a **coordinate system.** The number associated with a given point is called the **coordinate** of the point. The set of all real numbers is graphed in FIGURE 2.

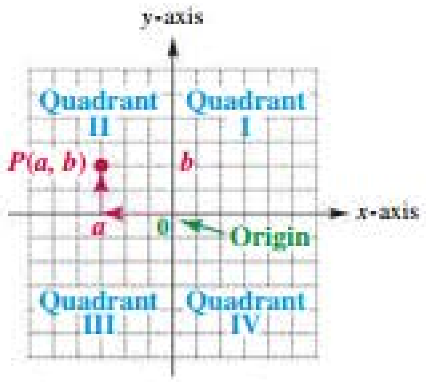
Irrational numbers cannot be represented by quotients of integers or by repeating or terminating decimals. Examples of irrational numbers include $\sqrt{3}$, $\sqrt{5}$, $\sqrt[3]{10}$, and $\sqrt[5]{20}$. If a is a natural number but $\sqrt{a}$ is not a natural number, then $\sqrt{a}$ is an irrational number. Another irrational number is π, which is approximately equal to 3.14159. In FIGURE 3 the irrational and rational numbers in the set $\left\{-\frac{2}{3}, 0, \sqrt{2}, \sqrt{5}, \pi, 4\right\}$ are located on a number line.



Graph of $\left\{-\frac{2}{3}, 0, \sqrt{2}, \sqrt{5}, \pi, 4\right\}$
FIGURE 3

The Rectangular Coordinate System

Two number lines at right angles, intersecting at their origins, form a two-dimensional **rectangular coordinate system.** This rectangular coordinate system is also called the Cartesian coordinate system, named for René Descartes (1596–1650). The number lines intersect at the *origin* of the system, designated 0. The horizontal number line is called the ***x*-axis,** and the vertical number line is called the ***y*-axis.** On the x-axis, positive numbers are located to the right of the origin, with negative numbers to the left. On the y-axis, positive numbers are located above the origin, with negative numbers below.

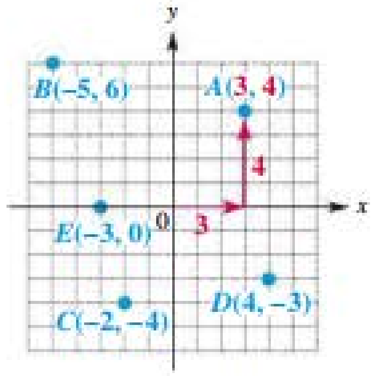


Rectangular coordinate system

FIGURE 4

The plane into which the coordinate system is introduced is the **coordinate plane,** or ***xy*-plane.** The x-axis and y-axis divide the plane into four regions, or **quadrants,** as shown in **FIGURE 4.** ***The points on the x-axis or y-axis belong to no quadrant.***

Each point P in the xy-plane corresponds to a unique ordered pair (a, b) of real numbers. We call a the ***x*-coordinate** and b the ***y*-coordinate** of point P. The point P corresponding to the ordered pair (a, b) is often written as $P(a, b)$, as in **FIGURE 4,** and referred to as "the point (a, b)." **FIGURE 5** illustrates how to plot the point $A(3, 4)$. Additional points are labeled B–E. The coordinates of the origin are $(0, 0)$.

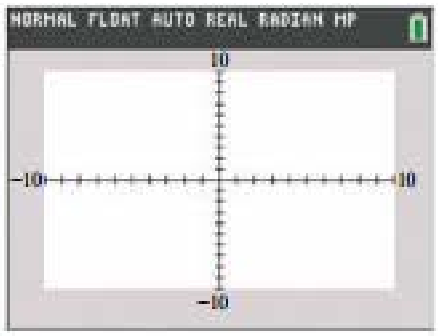


Plotting points in the xy-plane

FIGURE 5

Viewing Windows

The rectangular (Cartesian) coordinate system extends indefinitely in all directions. We can show only a portion of such a system in a text figure. Similar limitations occur with the viewing "window" on a calculator screen. **FIGURE 6** shows a calculator screen that has been set to have a minimum x-value of -10, a maximum x-value of 10, a minimum y-value of -10, and a maximum y-value of 10. The tick marks on the axes have been set to be 1 unit apart. This is the **standard viewing window.**



Standard viewing window

FIGURE 6

To convey information about a viewing window, we use the following abbreviations.

Xmin:	minimum value of x	**Ymin:**	minimum value of y
Xmax:	maximum value of x	**Ymax:**	maximum value of y
Xscl:	scale (distance between tick marks) on the x-axis	**Yscl:**	scale (distance between tick marks) on the y-axis

To further condense this information, we use the following symbolism, which gives viewing information for the window in **FIGURE 6.**

Xmin → Xmax → Ymin → Ymax

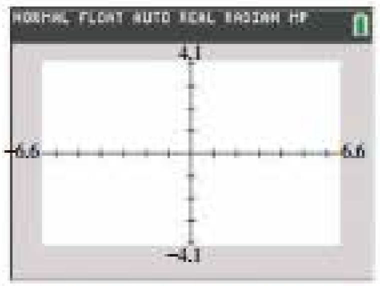
$$[-10, 10] \text{ by } [-10, 10]$$

Xscl = 1 Yscl = 1

FIGURE 7 shows several other viewing windows. Notice that **FIGURES 7(b)** and **7(c)** look exactly alike, and unless we know what the settings are, we have no way of distinguishing between them. In **FIGURE 7(b)** Xscl $= 2.5$, while in **FIGURE 7(c)** Xscl $= 25$. The same is true for Yscl in both.

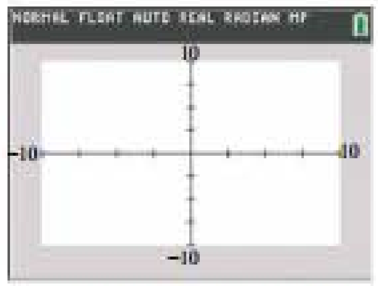
TECHNOLOGY NOTE

In this text we use screen captures from the TI-84 Plus C. You should consult your owner's guide to see how to set the viewing window on your screen. Remember that different settings will result in different views of graphs.



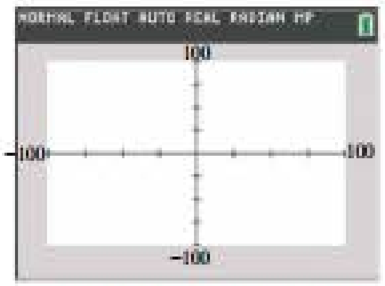
$[-6.6, 6.6]$ by $[-4.1, 4.1]$
Decimal viewing window

(a)



$[-10, 10]$ by $[-10, 10]$

(b)



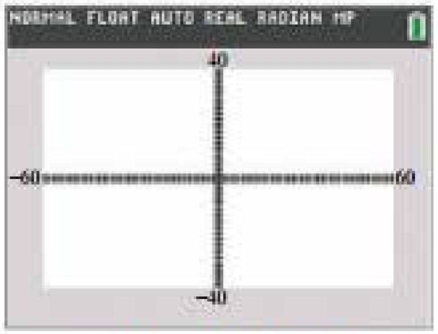
$[-100, 100]$ by $[-100, 100]$

(c)

FIGURE 7

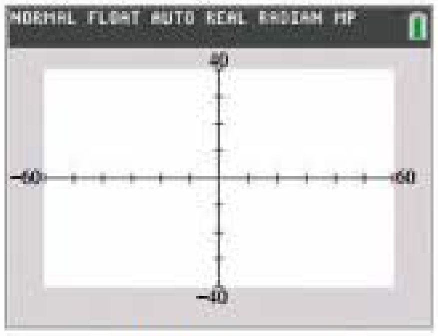
WHAT WENT WRONG?

A student learning how to use a graphing calculator could not understand why the tick marks on the graph were so close together, as seen in **FIGURE A,** while those on a friend's calculator were not, as seen in **FIGURE B.**



Xscl = 1, Yscl = 1

FIGURE A



Xscl = 10, Yscl = 10

FIGURE B

What Went Wrong? How can the student correct the problem in **FIGURE A** so that the axes look like those in **FIGURE B**?

TI-84 Plus C

FIGURE 8

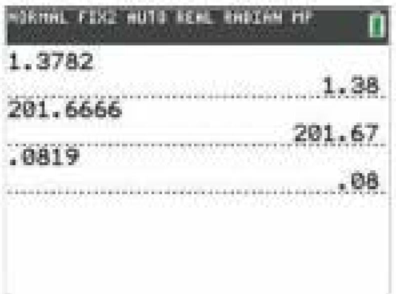


FIGURE 9

FIGURE 10

Approximations of Real Numbers

Although calculators have the capability to express numbers like $\sqrt{2}$, $\sqrt[3]{5}$, and π to many decimal places, we often require that answers be rounded. The following table reviews rounding numbers to the nearest tenth, hundredth, or thousandth.

Rounding Numbers

Number	Nearest Tenth	Nearest Hundredth	Nearest Thousandth
1.3782	1.4	1.38	1.378
201.6666	201.7	201.67	201.667
0.0819	0.1	0.08	0.082

In **FIGURE 8,** the TI-84 Plus C graphing calculator is set to round values to the nearest hundredth (two decimal places). In **FIGURE 9,** the numbers from the preceding table are rounded to the nearest hundredth.

The symbol $\approx$ indicates that two expressions are *approximately equal.* For example, $\pi \approx 3.14$, but $\pi \neq 3.14$ because $\pi = 3.141592653\ldots$. When using π in calculations, be sure to use the built-in key for π rather than 3.14. See **FIGURE 10.**

Answer to What Went Wrong?

Because Xscl = 1 and Yscl = 1 in **FIGURE A,** there are 120 tick marks along the x-axis and 80 tick marks along the y-axis. The resolution of the graphing calculator screen is such that these tick marks are nearly indistinguishable. The values for Xscl and Yscl need to be larger, equal to 10, as in **FIGURE B.**

EXAMPLE 1 Finding Roots on a Calculator

Approximate each root to the nearest thousandth. (*Note:* We can use the fact that $\sqrt[n]{a} = a^{1/n}$ to find roots.)

(a) $\sqrt{23}$ **(b)** $\sqrt[3]{87}$ **(c)** $\sqrt[4]{12}$

Solution

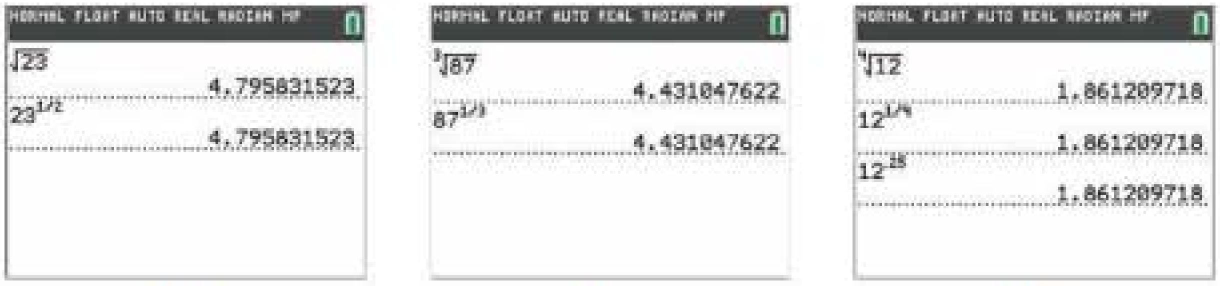
(a) The screen in FIGURE 11(a) shows an approximation for $\sqrt{23}$. To the nearest thousandth, it is 4.796. The approximation is displayed twice, once for $\sqrt{23}$ and once for $23^{1/2}$.

(b) To the nearest thousandth, $\sqrt[3]{87} \approx 4.431$. See FIGURE 11(b).

(c) FIGURE 11(c) indicates $\sqrt[4]{12} \approx 1.861$ in three different ways.

TECHNOLOGY NOTE

Graphing calculators have built-in keys for calculating square roots and menus for calculating other roots. The TI-84 Plus C has two print modes that will be used in this text: MATHPRINT and CLASSIC.



(a) (b) (c)

FIGURE 11

EXAMPLE 2 Approximating Expressions with a Calculator

Approximate each expression to the nearest hundredth.

(a) $\dfrac{3.8 - 1.4}{5.4 + 3.5}$ **(b)** $3\pi^4 - 9^2$ **(c)** $\sqrt{(4-1)^2 + (-3-2)^2}$

Solution

TECHNOLOGY NOTE

Graphing calculators may or may not display leading zeros in decimal numbers. For example, $\frac{1}{4}$ might be displayed as either 0.25 or .25.

(a) See the first display in FIGURE 12. To the nearest hundredth,

$$\frac{3.8 - 1.4}{5.4 + 3.5} \approx 0.27.$$

(b) To the nearest hundredth, $3\pi^4 - 9^2 \approx 211.23$. See the second display in FIGURE 12.

(c) From the third display in FIGURE 12, $\sqrt{(4-1)^2 + (-3-2)^2} \approx 5.83$.

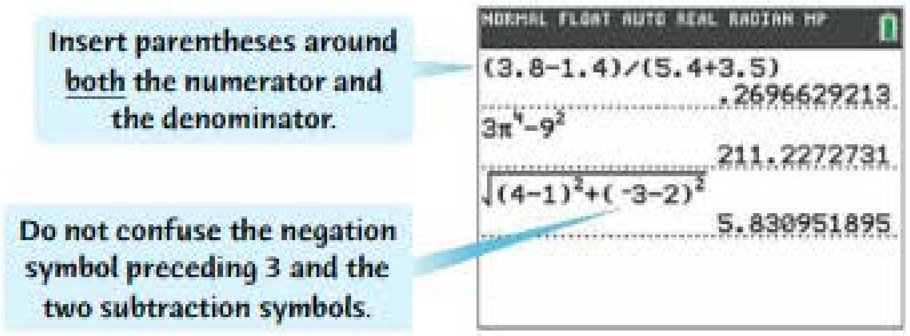


FIGURE 12

WHAT WENT WRONG?

Two students were asked to compute the expression $(2 + 9) - (8 + 13)$ on a TI-84 Plus C calculator. One student obtained the answer -10, as seen in **FIGURE A,** while the other obtained -231, as seen in **FIGURE B.**

FIGURE A

FIGURE B

What Went Wrong? Compute the expression by hand to determine which screen gives the correct answer. Why is the answer on the other screen incorrect?

Distance and Midpoint Formulas

The Pythagorean theorem is used to calculate lengths of the sides of a right triangle.

Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$

NOTE *The converse of the Pythagorean theorem is also true.* That is, if a, b, and c are lengths of the sides of a triangle and $a^2 + b^2 = c^2$, then the triangle is a right triangle with hypotenuse c. For example, if a triangle has sides with lengths 3, 4, and 5, then it is a right triangle with hypotenuse of length 5 because $3^2 + 4^2 = 5^2$.

EXAMPLE 3 **Using the Pythagorean Theorem**

Using the right triangle in **FIGURE 13,** find the length of the unknown side b.

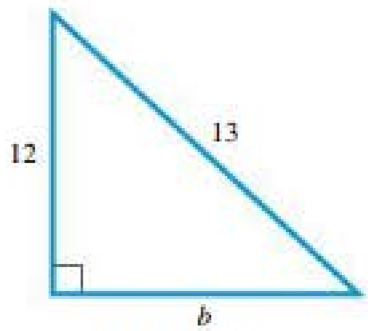


FIGURE 13

Solution Let $a = 12$ and $c = 13$ in the Pythagorean theorem.

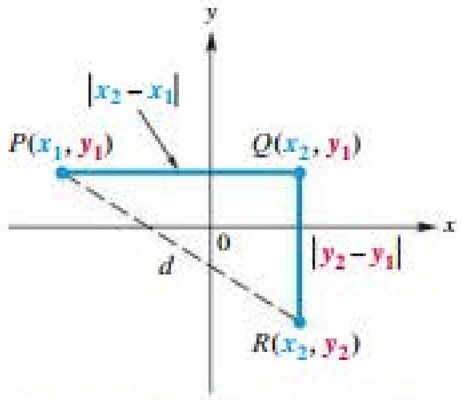
$$a^2 + b^2 = c^2 \quad \text{Pythagorean theorem}$$
$$12^2 + b^2 = 13^2 \quad \text{Substitute.}$$
$$b^2 = 13^2 - 12^2 \quad \text{Subtract } 12^2.$$
$$b^2 = 25 \quad \text{Simplify.}$$
$$b = 5 \quad \text{Take the positive square root.}$$

Answer to What Went Wrong?

The correct answer is -10, as shown in **FIGURE A. FIGURE B** gives an incorrect answer because the negation symbol is used, rather than the subtraction symbol. The calculator computed $2 + 9 = 11$ and then *multiplied* by the negative of $8 + 13$ (that is, -21), to obtain the incorrect answer, -231.

To derive a formula to find the distance between two points in the *xy*-plane, let $P(x_1, y_1)$ and $R(x_2, y_2)$ be any two distinct points in the plane, as shown in **FIGURE 14.** Complete a right triangle by locating point Q with coordinates (x_2, y_1). The Pythagorean theorem gives the distance between P and R as follows.

$$d(P, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



$d(P, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

FIGURE 14

NOTE Absolute value bars are not necessary in this formula because for all real numbers a and b, $|a - b|^2 = (a - b)^2$.

Distance Formula

Suppose that $P(x_1, y_1)$ and $R(x_2, y_2)$ are two points in a coordinate plane. Then the distance between P and R, denoted $d(P, R)$, is given by the **distance formula.**

$$d(P, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE 4 Using the Distance Formula

Use the distance formula to find the exact value of $d(P, Q)$ in **FIGURE 15.**

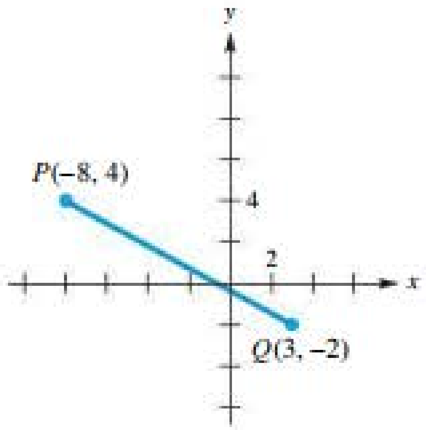


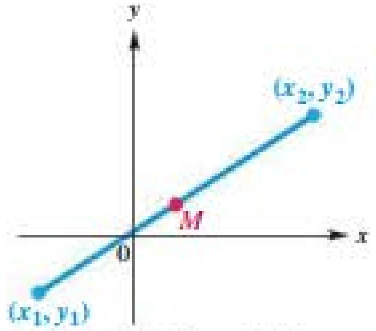
FIGURE 15

Solution

$$\begin{aligned} d(P, Q) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance formula} \\ &= \sqrt{[3 - (-8)]^2 + (-2 - 4)^2} && x_1 = -8,\ y_1 = 4,\ x_2 = 3,\ y_2 = -2 \\ &= \sqrt{11^2 + (-6)^2} \\ &= \sqrt{121 + 36} && \text{Apply exponents.} \\ &= \sqrt{157} && \text{Leave in radical form.} \end{aligned}$$

To subtract a negative number, add the opposite. That is, $3 - (-8) = 3 + 8$.

NOTE When using the distance formula, we usually leave the exact answer in radical form unless otherwise specified. Furthermore, we simplify the radical if possible. For instance, if the answer in **Example 4** were $\sqrt{169}$, we would simplify it as 13. If it were $\sqrt{156}$, we would simplify it as $\sqrt{4 \cdot 39} = 2\sqrt{39}$.



Point M is the midpoint of the segment joining (x_1, y_1) and (x_2, y_2).

FIGURE 16

The *midpoint* M of a line segment is the point on the segment that lies the same distance from both endpoints. See **FIGURE 16.** ***The coordinates of the midpoint are found by calculating the arithmetic mean (average) of the x-coordinates and the arithmetic mean of the y-coordinates of the endpoints of the segment.***

Midpoint Formula

The coordinates of the **midpoint** M of the line segment with endpoints (x_1, y_1) and (x_2, y_2) are given by the following.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$